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ABSTRACT

The effects of two curricula for teaching number concepts to kindergarten children: a part-part-whole (PPW) curriculum emphasizing set-subset relationships between cardinal numbers, and a count/say/write (CSW) standard curriculum were composed. The concept of number attained by children taught with these two curricula was compared to that of children who received no instruction in number. Data collection efforts involved evaluation of the children's understanding of the components of number, their problem-solving performance on addition and subtraction story problems, and their use of the PPW component of numbers with numbers greater than 10. Participants were 97 children from 5 kindergarten classes in a predominately white, middle-class public school. Findings indicated that the PPW instruction facilitated the development of number concept by kindergarten children. PPW also enhanced children's ability to solve addition and subtraction word problems. PPM may provide a conceptual base for emerging informal addition and subtraction strategies. Use of the CSW standard curriculum, even when carefully applied and upgraded to include many manipulative experiences, did not improve children's number concept. Children receiving no instruction performed better than did children receiving the CSW program of instruction. (RH)

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# A Comparison of Curricula used in Kindergarten for the Development of Number Concept

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# A Comparison of Curricula used in Kindergarten for the Development of Number Concept

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The set-subset relationship is an important mathematical construct that underlies the concept of number, the definition of addition and subtraction, the interpretation of complex addition and subtraction word problems, and the meaning of place value in the base-ten numeration system. The purpose of this study was to compare the effects, in the kindergarten, of a part-part-whole curriculum for teaching the concept of number which stressed set-subset relationships to the effects of a curriculum modeled after the standard curriculum. The concept of number attained by children in both of these curricula were also compared to that attained by children receiving no instruction in number. The children's attainment of concept of number was assessed by evaluating the children's understanding of the various components of number, their problem-solving performance on addition and subtraction story problems, and their use of the part-part-whole component of number with numbers greater than ten.

A component of number readily recognized by older but not younger children is the perception of a number not only as a whole but also as made up of its parts. Payne and Rathmell (1975) suggested the use of the words <u>whole</u> and <u>parts</u> to emphasize this essential partitioning relationship. A set of objects associated with a number can be separated into subsets,

or parts. Each of these subsets may also be represented by a number. For example, seven can be represented as {ooooooo} and also as {ooo/oooo} or {ooooo/oo}. Resnick (1983) stated that a major conceptual achievement in the early school years is the understanding of number as made up of parts.

This Part-Part-Whole (PPW) schema is increasingly recognized by educators as a necessary component of the ability to solve addition and subtraction problems. Riley, Greeno, and Heller (1983) have attempted to model addition and subtraction problem solving. In their model, it is not until the third, and last, level of performance that PPW relationships are established. Therefore, PPW is considered to be a more complex and mature way of interpreting addition and subtraction, freeing the child from the need to rely on direct concrete representations of the specific problem at hand. By enabling the child to see each number or quantity in a problem as associated with a set and also as associated with a subset, PPW allows the child to be more flexible in thinking, using mental counting strategies or derived fact strategies as needed. Further, PPW also allows the child to solve all basic addition and subtraction problems as the differing semantic structures may be interpreted in terms of parts and wholes. (Carpenter, 1986; Cobb, 1987; Romberg & Carpenter, 1986).

Part-part-whole instruction may also reduce the adjustments that children must make when first learning place value. Resnick (1983) recognized the importance of the PPW component of number, when she noted that PPW seemed to be a basic part of the

development of an understanding of place value, when children must learn to interpret numbers in terms of a "tens part" and a "ones part". This, followed by the coordination of counting by tens and by ones, is a strong indication of the need for the development of the PPW schema.

## The Curricula

The curricula used in this study concentrated on the cardinal component of number. Lessons were designed that reflected the two curricular approaches to the number concept. The number of activities, worksheets, and counting experiences provided for the children were controlled across the two curricula, as was the time spent in instruction.

### Count/Say/Write

The standard curriculum currently used in the kindergarten for the development of number concepts emphasizes counting, identifying, and writing the numeral that is associated with a set. This standard curriculum was characterized as Count/Say/Write (CSW), since most of the activities in it involve counting discrete sets, saying or recognizing the numbers associated with these sets, and then writing the corresponding numerals. The CSW curriculum used in this study was modeled after this standard curriculum but the curriculum was enhanced to include more manipulative activities.

### Part-Part-Whole

The Part-Part-Whole, PPW, curriculum developed for this study also focused on the cardinal component of number. However, this curriculum emphasized the use of the part-whole component of

number in developing cardinal concepts. Instead of children simply counting sets and saying or writing the numerals associated with them, the children explored the various parts into which the sets could be separated. For example, an activity designed to teach the concept of "fourness" would have the children investigate subsets by distributing, in various ways, a set of four objects into two containers. Manipulative materials were used in this exploration, and worksheets were developed to reinforce the concept.

## Method

A pretest-posttest design was used to assess the effect of the three treatments (PPW, CSW, and no instruction) on a sample of children (n=97) from five kindergarten classes in a predominately white, middle class public school. All classes were half-day sessions. Two of the classes, one morning and one afternoon, received the PPW curriculum, and two classes, one morning and one afternoon, received the CSW curriculum. The fifth class, morning, received no instruction in mathematics between the pretest and posttest. It was not possible to randomly assign the subjects to the classes; however, since the children were assigned haphazardly to the classes at the beginning of the year, lack of random assignment was not a major detriment to the study. However, classes were randomly assigned to treatments.

Of the 97 subjects, 54 were male and 43 were female. Forty children were instructed in the Part-part-whole curriculum, 42 in the Count/Say/Write curriculum, and 15 received no mathematics

instruction. The sample ranged in age from 58 to 71 months; the mean was 63.32 months.

## Procedure

The study was completed in the fall of the kindergarten year. The children were individually pretested. Instruction in classes receiving the PPW and the CSW curricula took place over the next 25 days. Each instructional session was approximately 20 minutes. The Control class received no special instruction. At the conclusion of the instructional period, each child was individually posttested. Instruction was done in the regular classrooms in both the CSW and PPW curricula by the same experienced teacher, not associated with the school. Periodic observations were made to assure the use of the curricula as planned and to document the number of counting experiences and the use of manipulatives in both curricula.

## Instruments

The children's conceptual sophistication in terms of various components of number (Number Concept Test), their performance on addition and subtraction story problems (Problem Solving Test), and their ability to use the part-part-whole interpretation of number with numbers greater than ten (Place Value Test) were used as the means of curriculum assessment. The three tests were administered in 20-25 minute individual interviews by an examiner who was naive as to the curriculum assigned to a child.

### Number Concepts Test

The Number Concepts Test (19 items) was developed to test nine components of number: rote counting, rational counting or

enumeration, cardinal number, subitizing, ordinal number, less than-greater than, one-to-one correspondence, conservation of number, and part-part-whole. The list of components was derived from various tests and commentary in the literature (Clements, 1984; Dimitrovsky & Almy, 1980; Elkind, 1961; Fuson & Hall, 1983; Gelman and Gallistel, 1978; Grossnickle & Bruechner, 1963; Kamii, 1980; Ogletree, Rackauskas & Buergin, 1970; Hendrickson, 1979; Piaget, 1953; Rea & Reys, 1970; Resnick, 1983; Saxe, 1977; Schaeffer, Eggleston & Scott, 1974; Silverman & Rose, 1980).

Rote counting (1 item). Counting forward from one was the task used to test rote counting. Each child was asked to count, and prompted if there was hesitation. Children that counted beyond twelve were scored correct on the rote counting item.

Rational counting (1 item). Each child was asked to count a randomly arranged set of six blocks.

Cardinal number (3 items). After counting the set of blocks in the rational counting task, the set was covered and the child asked, "How many blocks are on the table?" The second and third items used to test the cardinal component of number asked the child to make sets of a specified size (seven and twelve) by selecting objects from a larger set.

Subitizing (3 items). Prepared sets were made by picturing chips on cards (2, 4, and 6 chips). Each card was shown to the child for approximately one second. Then the card was placed out of sight. The child was then asked to state the number of chips that were shown on the card.

One-to-one correspondence (1 item). Given a set of seven

objects placed in a row on the table, the child was asked to make another row having the same number of objects. The objects in the child's row were of a different color from those used in the given row.

Conservation of number (2 items). Using the objects from the one-to-one correspondence task, equality of number was established. Then one set was moved so that the length of the row was longer than that of the other set and the child was asked if the two rows still had the same number. In a second task, one row of chips was counted, then the chips were rearranged into a circle. The child was then asked how many chips were in the circle. If counting was needed to answer the question asked after arrangement in each task, then the child did not demonstrate conservation of number.

Inequality (1 item). The child was asked to choose the more numerous of two sets of dots on a card.

Ordinal number (3 items). In the first item, the child was asked to point to the third car in a line of cars. In the second item, the children were asked to name the ordinal number for the red car in a row of blue ones. The last item required the child to place four randomly presented cards showing sets, of four, five, seven, and eight objects, in order from least numerous to most numerous.

Part-part-whole (4 items). In the first two items, two sticks of Unifix blocks were presented to the child. Each stick was composed to two colors of blocks in two different part-part-whole configurations. The child was requested to count each total stick and each of the parts, then the child was asked if

the two sticks had the same number of cubes. In the third item, a row of chips was presented to the child. The chips were counted, then some of the chips were turned over (the chips were a different color on the other side). The child was then asked, "How many chips are in the row?" The final item in this series of tasks asked for the number of one part when the whole and the other part were known.

Scoring. A total score on the Number Concepts Test was obtained by counting the points a child attained out of the 19 possible points.

Problem Solving Test

The six addition and subtraction problems used in this study were adapted from Carpenter's work (1985). The problem structure defined by Carpenter was preserved, but the situations were changed to use different contexts in each problem. Of the twenty types of problem structure listed by Carpenter (1985, p. 19), nine types were not used because they were reported to be either too easy or too hard for kindergarten children (Riley, 1981, reported by Carpenter, 1985). Equalize problems were also discarded, because of their difficulty. Therefore five types of problems were used: Join Result Unknown, Separate Change Unknown, Join Change Unknown, Separate Start Unknown, and Combine Subtraction. A sixth problem of the Combine Subtraction type was used to present a combine problem using only one type of object.

Scoring. For each of the six problems used in the Problem Solving Test, a child was given one point for a correct numerical answer and one point for a correct strategy regardless of the

numerical answer given, resulting in a possible twelve points on the test.

Place Value Test

The four items on the Place Value Test were adapted from beginning exercises for the development of place value concepts in standard textbooks. In the first task, the children were asked to count a set of 14 items and then to separate the set into a set of ten and a remainder set. A tray that would hold ten blocks was introduced in the second task. The children were to fill the tray from a set of 15 blocks, counting as they did so. Then the remaining blocks were counted, and the child was asked to state how many blocks there were in all. In the third task, the ten-tray was presented full of blocks. The children were asked to count the blocks in the tray and the loose blocks. Then they were asked how many blocks there were in all.

The last item in this test combined the place-value task with the part-part-whole component of number. The children were shown 13 blocks and an empty ten-tray. After counting the blocks, they were reminded that the tray would hold ten blocks, then asked, "If I filled the tray with some of the blocks, how many would not fit in the tray?" The child was allowed to move and count the blocks but was not allowed to have the ten-tray to fill.

Scoring. For each item, a point was given for the correct total number. An additional point was given if the child did not have to count from one to arrive at the correct number. For two of the items, points were also given for correctly counting the set of ten and the loose ones. There were 12 points possible on

the Place Value Test.

## Results

Collected data were analyzed using the SPSSX statistical package. First the pretest data were analyzed to determine if differences existed between treatment groups prior to instruction. Analysis of variance revealed no significant differences between treatment groups on any of the tests (Number Concepts Test: $F = 2.14$, $p = 0.12$; Problem Solving Test: $F = 1.71$, $p = 0.19$; Place Value Test: $F = 0.89$, $p = 0.42$).

In order to analyze the change in the subjects concept of number, gain scores were computed by subtracting the pretest score from the posttest score on each of the three tests for each subject. Multivariate analysis of variance was used to analyze the resulting gain scores. The results of this analysis indicated that there was a significant effect due to instructional treatment ($F = 7.07$; $p < 0.001$).

Because the multivariate analysis of variance indicated a significant difference in achievement of number concept, problem solving ability, and place value concepts, subsequent univariate analyses were performed. These analyses were performed on the difference scores (posttest - pretest).

Number Concepts Test. The results of the analysis of variance performed on the gain scores obtained from the pretest and posttest data on the Number Concepts Test indicate that there was a treatment effect ($F = 10.45$, $p = 0.0001$). Subsequent analysis using the Scheffe procedure revealed that the scores of the group receiving instruction in the part-part-whole curriculum

was significantly different from both the group receiving instruction in the count/say/write curriculum and the group receiving no instruction ($p < 0.05$). Examination of the means (See Table 1) reveals that the PPW group made greater gains during the instructional period than did the other groups. Additionally, there was no significant difference found between the CSW group and the Control group.

Problem Solving Test. In an univariate analysis, similar to the one used on the gain scores of the Number Concepts Test, the gain scores on the Problem Solving Test were analyzed. Again a significant treatment effect was found ($F = 13.52$, $p < 0.0001$). The use of the Scheffe procedure indicated that there was a significant difference ($p < 0.05$) between the PPW group and both the CSW group and the Control group. Examination of the means (see Table 1) revealed that the PPW group achieved a greater mean gain score than did the other groups. Also, the mean gain scores of the CSW group and the Control group were not significantly different.

Place Value Test. A third univariate analysis was performed on the gain scores obtained from the pretest and posttest data on the Place Value Test. A significant treatment effect was indicated ($F = 3.98$, $p = 0.02$). A subsequent use of the Scheffe procedure revealed that the gain scores of the PPW group were significantly different from the gain scores of the CSW group ($p < 0.05$). No other significant differences were observed. Examination of the means in Table 1 revealed that the mean gain score of the PPW group was higher than the mean gain score of the

CSW group. Although not significantly different, the mean gain score of the control group, receiving no instruction, was lower than that of the CSW group.

## Discussion and Conclusions

The PPW instruction facilitated the development of number concept by kindergarten children. It also enhanced the children's ability to solve addition and subtraction word problems and may provide a conceptual base for emerging informal addition and subtraction strategies. Additionally, early instruction in PPW may expediate instruction in place value and provide a continuity for interpretation of large and small numbers. However, the use of the standard curriculum, even when carefully applied and upgraded to include many manipulative experiences did not result in better development of number concept than did no instruction. In fact, children receiving no instruction performed better than did the children receiving this enhanced standard curriculum. This study provides evidence that kindergarten children can be taught the number concept if taught as PPW; but the standard curriculum, as it is now taught, may be no better than no instruction.

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Table 1

Means (standard deviations) of gain scores (posttest-pretest)

| | Treatment | | | |
|---|---|---|---|---|
| | PPW | CSW | Control | Combined |
| N | 40 | 42 | 15 | 97 |
| Number Concept Test | 5.2 (2.5) | 2.85 (2.46) | 3.27 (1.75) | 3.89 (2.61) |
| Problem Solving Test | 4.95 (2.79) | 1.86 (3.20) | 1.67 (2.47) | 3.10 (3.29) |
| Place Value Test | 3.58 (2.80) | 2.07 (2.32) | 2.33 (1.95) | 2.73 (2.56) |